Unique Paper Code : **32351102_OC**

Name of Paper : **C2 – Algebra**

Name of Course : **CBCS BSc. (H) Mathematics**

Semester : **I**

Duration : **3 hours**

Maximum Marks : **75**

Attempt any four questions. All questions carry equal marks.

1. Find the fifth root of the complex number $(\sqrt{3} + i)$ and represent them in the complex plane.

   Solve the equation $z^8 - z^5 + z^3 - 1 = 0$.

   Express $sin7\theta$ in terms of powers of $sin\theta$ and $cos\theta$.

2. For $a, b \in N$ define the relation $\sim$ on N by $a \sim b$ if and only if $a \div b = 2^k$ for some integer k.

   (i) Prove that $\sim$ defines an equivalence relation on N

   (ii) Find the equivalence class $\bar{1}$.

   Let $A = \{x \in R \mid x \neq 1\}$. Show that the function $f: A \to R$ defined by $f(x) = 4 + \frac{1}{x-1}$, is one to one. Also find the range and suitable inverse of $f(x)$.

   Show that the composition of two one to one functions is also one to one but the converse is not true.

3. Prove that the intervals $(2, \infty)$ and $(4, \infty)$ have the same cardinality. Express gcd (260,154) in the form $260x + 154y$, where x, y are integers. Find the value of $2^{100}(mod 11)$.

4. Describe all solutions of the following system in parametric vector form. Also give geometric description of the solution set.

$$2x + 2y + 2z = 0$$
$$-2x + 5y + 2z = 1$$
$$8x + y + 4z = -1$$

   Show that the following set of vectors in $\mathbb{R}^3$ is linearly dependent.

$$\left\{ \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -5 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$$

5. $\beta = \{b_1, b_2, b_3\}$ is an ordered basis for R^3 where $b_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $b_3 = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$.

For $v = \begin{bmatrix} 0 \\ -1 \\ 7 \end{bmatrix}$ find $[v]_\beta$,the coordinate vector of v relative to the ordered basis β.

Find the basis and dimensions of Col(A) and Nul(A) for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$$

Also verify Rank - Nullity theorem for the Matrix A.

If the standard matrix of a linear transformation T: $R^4 \to R^3$

is the matrix A (given above) find T $\begin{bmatrix} 1 \\ -1 \\ 2 \\ 3 \end{bmatrix}$

6. Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$

Show that $\lambda = -2$ is an eigen value of A and find one eigen vector of A corresponding to $\lambda = -2$

T: $R^3 \to R^3$ is a linear transformation such that
T $(x_1, x_2, x_3) = (x_2 + 4x_3, x_1 + 2x_2 - x_3, 5x_1 + 8x_2)$

(i) Find the standard matrix of T
(ii) Is T one to one? Is T onto? Is T invertible? Justify your answer.
(iii) Find x such that $T(x) = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$